UDC 551.513.1:551.511.6(213

ANALYTIC MODEL FOR ZONAL WINDS IN THE TROPICS

I. Details of the Model and Simulation of Gross Features of the Zonal Mean Troposphere [1]

ROBERT E. DICKINSON

National Center for Atmospheric Research,[2] Boulder, Colo.

ABSTRACT

We consider the zonal mean circulation on an equatorial β-plane and obtain analytic solutions for small-amplitude disturbances in response to sources of heat and momentum. The effect of dissipation is roughly approximated by using assumed constant drag and radiative damping terms. We apply the model to the tropospheric zonal winds, temperatures, and meridional circulation to give simple insights into the maintenance of the observed mean state of these parameters by the existing sources of heat and momentum. The heat source consists of a relatively sharp maximum in heating at the Equator caused by latent heat release in the tropical rainbelt against a relatively smooth background of mean radiative cooling. As a momentum source, we use the convergence of horizontal eddy momentum fluxes. Numerical results are presented for an idealized annual mean circulation. The model solutions show how the thermal and momentum sources jointly maintain the zonal wind distribution. A concomitant meridional circulation redistributes the heat and momentum, allowing the temperatures to remain in balance with the zonal wind. This is part I of a two-part study.

1. INTRODUCTION

One of the goals of dynamical climatology is to understand the basic features of the zonal mean winds, temperatures, and meridional circulation and their variation with seasons and between hemispheres. The zonal mean state must necessarily be determined by the distribution of zonal mean sources of heat and momentum. Of those sources, only incoming solar radiation is entirely independent of the zonal mean variables. The calculation of all other atmospheric parameters, given the incident solar radiation and the governing equations, is attempted in the general circulation model experiments. Unfortunately, the relationships between various components of the circulation deduced in this fashion are extremely complicated; and therefore, understanding the detailed relationships between variables is difficult.

We are particularly interested in such questions as: (1) What are the relative roles of thermally forced Hadley circulation and eddy momentum transports in determining tropospheric zonal winds and mean meridional circulation? (2) To what extent and through what mechanisms does the vertical and latitudinal variation of temperature in the troposphere and lower stratosphere depend on dynamical rather than radiative balances?

To simplify the equations sufficiently to allow us some insight into the answers to these questions and others, we must omit many processes of secondary importance and refer to observations for some of the important parameters that must actually depend on the zonal mean state. The simplest viewpoint (Eliassen 1952, Kuo 1956) is that all components of the transfer of heat and momentum to the zonal mean state are known. The consequent meridional circulation is then to be determined. In this formulation, however, the sources of heat and momentum cannot be independently specified if the zonal wind is independent of time (Gilman 1964). For example, the net zonal mean momentum input balancing the Coriolis torque determines a mean meridional circulation. The consequent adiabatic heating must then be balanced by nonadiabatic heat loss of the same magnitude, or else a temperature rate of change inconsistent with a steady zonal wind is required.

The approach used here is to relate part of the momentum source distribution to the zonal wind as a frictional drag and part of the heat source distribution to the zonal temperature as a radiative cooling. We obtain as a function of the distribution of sources not only the meridional circulation but also the zonal wind and temperature perturbation. We introduce these two further variables and only one further equation (i.e., the hydrostatic equation), so the number of equations and dependent variables are the same and the remaining components of the momentum and heat sources can be independently specified even when they are independent of time.

Leovy (1964) developed a model of this type to examine the seasonal variation of the stratospheric zonal winds. Dickinson (1968) used a constant Coriolis parameter model to examine the various possible classes of motion when radiative, but not frictional, damping is assumed. Holton (1968) developed a numerical equatorial β-plane model that allows realistic inclusion of sources and nonlinear terms. He has used this model for studies of the biennial oscillation.

The present study uses a simplified linear version of the equatorial β-plane model to study the response of

[1] Preliminary results from this study were presented at the Royal Meteorological Society, American Meteorological Society, Joint Conference on the Global General Circulation, London, August 1969.

[2] The National Center for Atmospheric Research is sponsored by the National Science Foundation.

equatorial zonal wind, temperature, and Hadley circulation to sources of heat and momentum in the troposphere. We are here particularly interested in understanding the coupling between the tropical and extratropical symmetric circulation. The latitudinal and vertical dependence of the prescribed heating and momentum transports is given by functions, described in the next section, that facilitate analytic solution of the zonal mean equations and have roughly the same amplitudes and spatial distribution as the actual sources.

It is now generally recognized that the primary thermal source in the Tropics is the release of latent heat in tall cumulus towers along the intertropical convergence zone. Several ways to parameterize this process in terms of the motions have been suggested; but none is readily adapted to a linear model for motions on a seasonal time scale, so that the heat input associated with the tropical rain belt is regarded here as a known function of latitude, pressure, and time.

Our discussion proceeds as follows: First, we formulate the theoretical model for zonal winds and obtain its analytic solution. Then, we describe numerical results obtained by evaluating the solution, using a standard set of parameter values, to determine the model response to an idealized annual mean forcing. Finally, we indicate the nature of the response for the various possible limiting cases.

The present study shows gross features of the annual mean symmetric circulation (tropical and mid-latitude) to be qualitatively reproduced by the simple version of the model herein derived. In part II, a companion paper (Dickinson 1971), we discuss simulation of seasonal changes and hemispheric differences of the zonal mean troposphere, using a somewhat more complicated version of the present model.

2. FORMULATION AND SOLUTION OF THE MODEL

GOVERNING EQUATIONS

The basis for our analysis is the following system, eq (1) to (4), for the zonal circulation on an equatorial β-plane. Bars denote zonal averages, primes denote deviations from the zonal average, u is zonal wind, v is meridional wind, T is the deviation from some hemispheric mean radiative equilibrium temperature T_0, h is deviation from a hemispheric mean geopotential height, z is log (p_0/p) where p is pressure and p_0 is a reference pressure, $w=\dot{z}$, and r_e is the radius of the earth. The sine of latitude is the nondimensional meridional coordinate $y, f=2\Omega y$, R is the gas constant, c_p is specific heat, and Q is the rate of heat addition per unit mass. In the calculations to be discussed, α is a constant Newtonian cooling coefficient; and d is a constant drag coefficient. We can still obtain eq (6) as we generalize the formulation by regarding either: (1) d and α as constant-coefficient differential operators in the variable z or (2) d as a constant parameter but α as a rather general z-dependent operator as described by Dickinson and Geller (1968). The equations are written as

$$\frac{\partial\bar{u}}{\partial t}+d\bar{u}-f\bar{v}=-\frac{1}{r_e}\frac{\partial}{\partial y}\overline{u'v'}, \tag{1}$$

$$f\bar{u}+\frac{g}{r_e}\frac{\partial\bar{h}}{\partial y}=0, \tag{2}$$

$$g\frac{\partial\bar{h}}{\partial z}=R\bar{T}, \tag{3}$$

and

$$\frac{\partial\bar{T}}{\partial t}+\alpha\bar{T}+\left(\frac{\partial T_0}{\partial z}+\frac{R}{c_p}\ T_0\right)\bar{w}=\frac{\bar{Q}}{c_p}. \tag{4}$$

The system is completed by the continuity equation in pressure coordinates, which is used to introduce the stream function ψ; thus,

$$\bar{v}=e^z\partial\psi/\partial z, \quad \bar{w}=-e^z r_e^{-1}\partial\psi/\partial y. \tag{5}$$

Our geometry is unbounded laterally and above. Vertical motion vanishes at the bottom boundary. The solutions need only satisfy certain integrability requirements for large values of $|y|$ and z. These solutions (roughly speaking) decay to zero at great heights and at large values of y. If α and d are to be considered z-dependent operators, further conditions are needed at the lower boundary.

By some obvious differentiations, eq (1) to (5) reduce to a single partial differential equation in the stream function ψ, which is written as

$$\left(\frac{\partial}{\partial t}+\alpha\right)\left(\frac{\partial^2\psi}{\partial z^2}+\frac{\partial\psi}{\partial z}\right)+\left(\frac{\partial}{\partial t}+d\right)\frac{S}{y^2}\frac{\partial^2\psi}{\partial y^2}=F \tag{6}$$

where F is given by

$$F=\frac{e^{-z}/y^2}{2\Omega r_e}\left[y\left(\frac{\partial}{\partial t}+\alpha\right)\frac{\partial^2\overline{u'v'}}{\partial y\partial z}-\frac{\kappa}{2\Omega}\left(\frac{\partial}{\partial t}+d\right)\frac{\partial\bar{Q}}{\partial y}\right].$$

Here, $\kappa=R/c_p$; and $S(z)=(\kappa T_0+\partial T_0/\partial z)/(2\Omega r_e)^2$ is a nondimensional static stability.

PHYSICAL BASIS FOR PARAMETERIZATION OF THE DISSIPATION AND FORCING TERMS

The Newtonian cooling approximation neglects the transfer of heat between atmospheric layers and assumes the net exchange of a layer with the ground and space is linear in the temperature perturbation. The Rayleigh friction approximation neglects the frictional exchange between layers and assumes the loss of momentum by a layer to the ground is proportional to the velocity of that layer.

To specify more carefully the heat input Q and the Newtonian cooling in eq (4) would require that the heat balance computed using the parameterization of this paper be a "best fit" to the heat balance of some realistic model for the troposphere. Such a model should include not only infrared cooling but also other highly temperature-dependent processes such as heat transport by small- and large-scale eddies and effects of temperature-dependent cloudiness and water vapor content. First, we would have to obtain a global mean temperature $T_0(z)$ which gave global mean radiative cooling that balances the global mean heating. The deviations from this global mean net heating of zero would then be expressed as a part essentially independent of the perturbation variables, a part proportional to the deviation of the temperature from the global mean temperature, and finally a part proportional to the mean vertical motion. The first part then yields Q, and the second part yields the Newtonian cooling coefficient α. The third part is our adiabatic cooling term in eq (4) and is to be predicted by the dynamics.

Without such an elaborate calculation, it is possible only to estimate α and Q roughly. The Newtonian cooling coefficient determines the rate of destruction of zonal available potential energy (ZAPE) and hence can be estimated from the global rate of ZAPE generation (Wiin-Nielsen et al. 1967). Time dependent radiative-balance calculations (e.g., Manabe and Strickler 1964) yield rates at which equilibrium temperatures are restored and which can be roughly equated to the appropriate Newtonian cooling coefficient. Both these considerations indicate that a reasonable value for α is (20 day)$^{-1}$.

Our assumed thermal source models the distribution in the tropical rain belt of latent heat release in excess of that necessary to balance the global mean rate of cooling. The latitudinal distribution of heating consists of a relatively sharp positive peak in the Tropics and a relatively flat negative region away from the Tropics. No attempt is made here to include details in the region of heat deficit resulting from such features as latitudinal variation of solar radiation and cloudiness, poleward eddy heat transports, and the secondary rainfall maximum in middle latitudes. We are interested in seeing what features of the zonal winds and temperatures can be ascribed to the excess heating in the tropical rain belt in conjunction with eddy momentum transports.

The heating is distributed in the vertical according to the hypothesis of Riehl and Malkus (1958), that the release of latent heat in the Tropics occurs primarily in "hot cumulus towers" penetrating from the surface boundary layer through the stable middle layers into the upper tropospheric layers. The precise vertical distribution of heating provided by these cumulus towers is unknown but cannot be too different from that predicted by the model of Kuo (1965). In Kuo's model, downdrafts and effects of entrainment are neglected so the heating is proportional to the difference between the temperature of a parcel that has ascended moist-adiabatically from the surface layer and the temperature of the ambient atmosphere. Our vertical distribution of heating is chosen to be maximum at around 300 mb and smoothly goes to zero at the surface as well as at the tropopause, as suggested by Kuo's figure 2.

The zonal mean momentum sinks of a realistic model in principle can also be divided, for a given spatial distribution of the momentum sinks, into a part that is linear in the mean flow (Rayleigh friction) and a remainder to be regarded as a known source term. The momentum exchange with the surface for a given wind strength is likely to be much greater in the lower layers of the troposphere than in higher layers. We here obtain a suitable mean value of the Rayleigh friction coefficient from observational estimates of the rate of frictional dissipation of kinetic energy. Oort (1964) gives values for zonal kinetic energy dissipation in the Northern Hemisphere; and Kung (1967), for total kinetic energy dissipation over North America. Both studies support the choice again of (20 day)$^{-1}$ as a mean damping time for zonal kinetic energy.

METHOD OF SOLUTION—SEPARATION OF VARIABLES

We are interested in solutions that are periodic in time, so both the forcing terms and the dependent variables are assumed proportional to $\exp[i\nu t]$, where ν is the frequency of the forcing. We then solve a two-dimensional boundary value problem in y and z. The problem is separable because of the linearization of eq (1) and (4) and because the boundary conditions are applied on a constant z surface and are independent of y.

It is convenient to express the stream function ψ as a linear combination of the eigensolutions to the Sturm-Liouville system

$$\frac{\partial^2\psi}{\partial y^2}+\lambda^2y^2\psi=0 \tag{7}$$

where λ is an eigenparameter. The forcing on the right-hand side of eq (6) is similarly taken proportional to a linear combination of solutions to eq (7) so that we separate from eq (6) the vertical structure equation

$$\frac{\partial^2\psi}{\partial z^2}+\frac{\partial\psi}{\partial z}-\tilde{a}^2\psi=(\alpha+i\nu)^{-1}f(z) \tag{8}$$

where $\tilde{a}^2=\lambda^2S(i\nu+d)/(i\nu+\alpha)$ and $f(z)$ is the projection of the right-hand side of eq (6) on that eigensolution of eq (7) with eigenvalue λ.

To simplify further the procedure for the separation of variables, we model the heating and momentum transports by the product of nondimensional functions of latitude with functions of z and with constant dimensional factors Q_c and M_c. We use subscripts c for constants, e and o for even and odd functions of latitude, and v for functions of the vertical coordinate

$$\left.\begin{aligned}Q(y,z)&=Q_cQ_v(z)(Q_e(y)+Q_o(y))\\ \overline{u'v'}(y,z)&=M_cM_v(z)(M_e(y)+M_o(y))\end{aligned}\right\}. \tag{9}$$

We now proceed to describe eigensolutions to eq (7) for the horizontal structure and then the inhomogeneous solutions to eq (8) for the vertical structure. Finally, we show how these are synthesized to provide a solution to the original system, eq (1) to (5).

HORIZONTAL STRUCTURE EIGENSOLUTIONS

The eigenvalue problem, eq (7), is solved for all real positive values of λ by an odd and an even power series. These are denoted, respectively, $\Psi_o(\lambda,y)$ and $\Psi_e(\lambda,y)$:

$$\left.\begin{aligned}\Psi_o(\lambda,y)&=y\left[1-\frac{(\lambda y^2)^2}{4\cdot 5}+\frac{(\lambda y^2)^4}{4\cdot 5\cdot 8\cdot 9}-\frac{(\lambda y^2)^6}{4\cdot 5\cdot 8\cdot 9\cdot 12\cdot 13}+\dots\right]\\ \Psi_e(\lambda,y)&=1-\frac{(\lambda y^2)^2}{3\cdot 4}+\frac{(\lambda y^2)^4}{3\cdot 4\cdot 7\cdot 8}-\frac{(\lambda y^2)^6}{3\cdot 4\cdot 7\cdot 8\cdot 11\cdot 12}+\dots\end{aligned}\right\}. \tag{10}$$

These functions are antisymmetric and symmetric, respectively, about the Equator. We also need the power series obtained by the differentiation of the above series. These are denoted $\Phi_e(\lambda,y)$, $\Phi_o(\lambda,y)$:

$$\Phi_e(\lambda,y)=\partial\Psi_o(\lambda,y)/\partial y,\ \ \Phi_o(\lambda,y)=\partial\Psi_e(\lambda,y)/\partial y. \tag{11}$$

From eq (7), we have the following additional identities:

$$\Psi_o=-(\lambda y)^{-2}\frac{\partial\Phi_e}{\partial y},\ \ \Psi_e=-(\lambda y)^{-2}\frac{\partial\Phi_o}{\partial y}. \tag{12}$$

Various properties of the series (9) and (10) and, in particular, their use in integral transforms can be deduced from their relationship to Bessel functions

$$\left.\begin{aligned}\Psi_o(\lambda,y)&=C_o\left(\frac{2y^2}{\lambda}\right)^{1/4}J_{1/4}(\lambda y^2/2)\\ \Psi_e(\lambda,y)&=C_e\left(\frac{\lambda y^2}{2}\right)^{1/4}J_{-1/4}(\lambda y^2/2)\end{aligned}\right\} \tag{13}$$

where the constants C_o and C_e are defined in terms of gamma functions $C_o=\frac{1}{4}\,\Gamma\,(\frac{1}{4})$, $C_e=\Gamma\,(\frac{3}{4})$.

We introduce the following Hankel transforms as applied to an odd function $F_o(y)$ or an even function $F_e(y)$:

$$F_o^*(\lambda)=C_o^{-2}(\lambda/2)^{3/2}\int_0^\infty y'^2\Psi_o(\lambda,y')F_o(y')dy' \tag{14a}$$

and

$$F_e^*(\lambda)=C_e^{-2}(\lambda/2)^{1/2}\int_0^\infty y'^2\Psi_e(\lambda,y')F_e(y')dy'. \tag{14b}$$

With these definitions, the inverse transforms are simply

$$F_o(y)=\int_0^\infty \Psi_o(\lambda,y)F_o^*(\lambda)d\lambda \tag{15a}$$

and

$$F_e(y)=\int_0^\infty \Psi_e(\lambda,y)F_e^*(\lambda)d\lambda. \tag{15b}$$

For the calculations discussed here in part I, we assumed the heating to be symmetric and the eddy momentum transport to be antisymmetric about the Equator. The procedure used was to "guess" appropriate simple analytic formulas for the transforms of sources. These formulas had several parameters that were adjusted by trial and error to obtain source shapes resembling those observed. The horizontal dependence of the even part of the heating Q is given by

$$Q_e(y)=\int_0^\infty \Phi_e(\lambda,y)\tilde{F}(\lambda)d\lambda \tag{16}$$

where we assumed for the calculations

$$\tilde{F}(\lambda)=\mathrm{I}^{-1}\exp\left[-2\cdot 10^{-4}\lambda^2+2e^{-0.05\lambda}\right] \tag{17}$$

and where I is the integral of $F_e(\lambda)$ from 0 to ∞, used as a normalization factor so that $Q_e(0)=\Phi_e(\lambda,0)=1$. Application of $y^{-2}\partial/\partial y$ to eq (16) and use of eq (12) reduce eq (16) to the form of eq (15a) where we identify $F_o^*(\lambda)$ with $-\lambda^2\tilde{F}(\lambda)$.

The antisymmetric horizontal structure of eddy momentum transports is synthesized using

$$M_o(y)=\int_0^\infty \mu_o(\lambda,y)\tilde{G}_o(\lambda)d\lambda \tag{18}$$

where $\mu_o(\lambda,y)$ is the power series solution obtained by integrating $y^{-1}\mu_o'(\lambda,y)=\Psi_o(\lambda,y)$. For the calculations, the spectral weighting function $\tilde{G}_o$ was taken to be

$$\tilde{G}(\lambda)=\sum_{j=1}^{4}C_j\mathrm{I}_j^{-1}\exp\left[-a_j\lambda^2-b_j/\lambda^2\right]+C_5\mathrm{I}_5^{-1}[(\lambda-\lambda_M)^2+d^2]^{-1} \tag{19}$$

where the (I_j)'s are normalization factors obtained as in eq (17). We assume values $\lambda_M=70$, $d=20$, $C_1=-1$, $C_2=3$, $C_3=16$, $C_4=11.2$, $C_5=24$, $a_1=10$, $a_2=0.01$, $a_3=0.002$, $a_4=0.05$, $b_1=1{,}000$, $b_2=1{,}000$, $b_3=3{,}000$, and $b_4=10^4$. The (C_j)'s are multiplied by 100 $\mathrm{m^2/s^2}$ to obtain dimensional momentum transports. We would need only one or two terms to adequately represent the eddy momentum transports for part I, but additional terms have been introduced here for correspondence with those used in obtaining seasonal and hemispheric changes of the transport in part II.

SOLUTION OF THE INHOMOGENEOUS VERTICAL STRUCTURE EQUATION

The horizontal dependence of the sources assumed separable according to eq (9) is expressed as integrals over the eigenparameter λ of the power series just described. After the stream function ψ is likewise expanded, we arrive at the separated eq (8)

$$f(z)=e^{-z}\left[K_1\frac{\partial}{\partial z}M_o(z)+K_2Q_o(z)\right] \tag{20}$$

where K_1 and K_2 are dimensional constants. The actual constants necessary for solutions with the appropriate physical dimensions will be given in the next section. It is sufficient here to consider the response to the nondimensional forcing $M_v(z)$ and $Q_v(z)$.

If we wished to model great detail in the vertical structure of the sources, static stability, and Newtonian cooling, we could readily solve eq (8) by finite differences. In view of our present ignorance of such detail and of the crudity of the separability approximation (9), such an approach is not justified here. Rather, we assume simple shapes roughly modeling the actual variation of the sources with height. By varying the parameters on which these shapes depend, we gain some insight into the sensitivity of our results to the details of the height variation of the sources.

We restrict ourselves for the remainder of this first paper to using constant values of S, d, and α. (The generalization to a stratosphere with large static stability is described in part II.) In the absence of a bottom boundary, eq (8) then has the general solution

$$\psi=-\frac{1}{2a}\int_{-\infty}^{\infty}\exp\left[-\tfrac{1}{2}(z-z')-a|z-z'|\right]f(z')dz' \tag{21}$$

using

$$a=[\tilde{a}^2+1/4]^{1/2}=[\lambda^2 S(i\nu+\alpha)^{-1}(i\nu+d)+1/4]^{1/2}. \tag{22}$$

To satisfy the bottom boundary condition, we must add to eq (21) a solution of the homogeneous equation that decays upward and in which the sum with eq (21) satisfies the appropriate bottom boundary condition at $z=0$:

$$\psi=0. \tag{23}$$

Let z_m denote the level of maximum heating and assume all the heating takes place within a depth of 2 D. The heat source used for our calculations is then written as

$$Q_v(z)=0, \quad \text{if } |z-z_m|>D$$

and

$$Q_v(z)=\exp[\tfrac{1}{2}(z-z_m)][1-|z-z_m|/D], \quad \text{if } |z-z_m|<D. \tag{24}$$

This function (fig. 1) models a source increasing from zero value to a maximum value, then decreasing to zero again with further increase of height. Adjustable parameters are its strength at the center z_m and half width D.

Substituting eq (24) into eq (21) and adding the proper homogeneous solution, we find

$$\psi(z)=-(a^3D)^{-1}\exp[-\tfrac{1}{2}(z+z_m)][W(\lambda,z)+W_I(\lambda,z)] \tag{25}$$

where $W(\lambda, z)$ is proportional to the solution forced by the source according to eq (21) and $W_I(\lambda, z)$ is the "image" solution added so that eq (23) is satisfied, that is,

$$W(\lambda,z)=[\cosh(aD)-1]\exp[-a|z-z_m|] \quad \text{if } |z-z_m|>D, \tag{26}$$

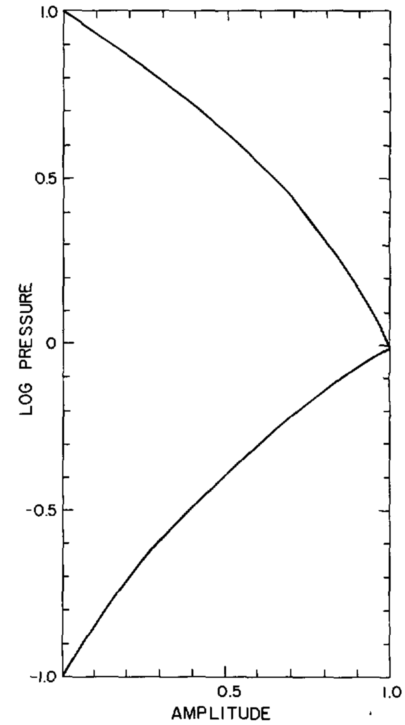

FIGURE 1.—Assumed shape for the vertical variation with height of the heating and eddy momentum flux.

$$W(\lambda,z)=e^{-aD}\cosh[a(z-z_m)]-e^{-a|z-z_m|}+a(D-|z-z_m|), \quad \text{if } |z-z_m|<D, \tag{27}$$

and

$$W_I(\lambda,z)=[\cosh(aD)-1]\exp[-a|z+z_m|]. \tag{28}$$

The momentum source $M_v(z)$ is also assumed to follow the vertical profile defined by eq (24) except that the center z_m and the half width D are not necessarily the same for $M_v(z)$ as for $Q_v(z)$. The solution to eq (8) in response to $M_v(z)$ is

$$e^z\psi=-\frac{1}{a^3D}\frac{\partial}{\partial z}\left\{e^{(1/2)(z-z_m)}\left[W(\lambda,z)+\frac{a+(1/2)}{a-(1/2)}W_I(\lambda,z)\right]\right\} \tag{29}$$

where $W(\lambda, z)$ and $W_I(\lambda, z)$ are again the expressions given by eq (26), (27), and (28).

SYNTHESIS OF THE TWO-DIMENSIONAL SOLUTIONS

We now describe the solutions obtained for the system (1) to (5) by solving eq (6) in terms of sources given by eq (9). The horizontal structure of the assumed sources is described by eq (16) to (19). The vertical structure of the heating and of the eddy momentum flux is given by eq (24). The two-dimensional structure of the dependent variables ψ, u, v, w, h, and T is then determined by integrating over λ the product of the following factors: the appropriate weighting function, one of the power series $\Psi_e(\lambda,y)$, $\Psi_o(\lambda,y)$, $\Phi_e(d,y)$, $\Phi_o(\lambda,y)$ defined by eq (10) and

(11), a dimensional factor, and derivatives of $W(\lambda,z)$ and $W_I(\lambda,z)$.

The response to the heating is then

$$\begin{bmatrix}(\psi,u,v)\\(w,h,T)\end{bmatrix}=\int_0^\infty \tilde{F}(\lambda)\exp\left[\tfrac{1}{2}(z-z_m)\right]\begin{bmatrix}\Psi_o(\lambda,y)\mathcal{L}_1^Q(\lambda,y,z)\\ \Phi_e(\lambda,y)\mathcal{L}_2^Q(\lambda,y,z)\end{bmatrix}\times(W(\lambda,z)+W_I(\lambda,z))d\lambda \tag{30}$$

where $\mathcal{L}_1^Q(\lambda,y,z)$ and $\mathcal{L}_2^Q(\lambda,y,z)$ are row matrices consisting of dimensional factors and involving the differential operators $\mathcal{D}=[(1/2)-(\partial/\partial z)]$ and $\mathcal{D}_2=[(1/4)-(\partial^2/\partial z^2)]$:

$$\left.\begin{aligned}\mathcal{L}_1^Q(\lambda,y,z)&=(-e^{-z}\Psi_D(\lambda),\ yU_D(\lambda)\mathcal{D},V_D(\lambda)\mathcal{D})\\ \mathcal{L}_2^Q(\lambda,y,z)&=(W_D(\lambda),H_D(\lambda)\mathcal{D},T_D(\lambda)\mathcal{D}_2)\end{aligned}\right\}. \tag{31}$$

The dimensional factors, proportional to the maximum heating Q_c, are written

$$\left.\begin{aligned}&\Psi_D=\frac{\lambda^2}{a^3D}\left(\frac{i\nu+d}{i\nu+\alpha}\right)\frac{\kappa Q_c}{4\Omega^2 r_e},\ V_D=\Psi_D,\ U_D=\left(\frac{2\Omega}{i\nu+d}\right)\Psi_D\\ &W_D=r_e^{-1}\Psi_D,\ H_D=\frac{\kappa Q_c}{a^3gD(i\nu+\alpha)},\ T_D=gH_D/R\end{aligned}\right\}. \tag{32}$$

For computing the response to the assumed eddy momentum transport, we use

$$\begin{bmatrix}(\psi,u,v)\\(w,h,T)\end{bmatrix}=\int_0^\infty \tilde{G}(\lambda)\exp\left[\tfrac{1}{2}(z-z_m)\right]\begin{bmatrix}\Psi_o(\lambda,y)\mathcal{L}_1^m(\lambda,y,z)\\ \Phi_e(\lambda,y)\mathcal{L}_2^m(\lambda,y,z)\end{bmatrix}\times\left(W(\lambda,z)+\left(\frac{a+\frac{1}{2}}{a-\frac{1}{2}}\right)W_I(\lambda,z)\right) \tag{33}$$

where if $\mathcal{D}^*=[(\partial/\partial z)+(1/2)]$, and again $\mathcal{D}_2=[(\partial^2/\partial z^2)-(1/4)]$, then

$$\left.\begin{aligned}\mathcal{L}_1^m&=(-e^{-z}\psi_c\mathcal{D}^*,yU_c,V_c\mathcal{D}_2)\\ \mathcal{L}_2^m&=(W_c\mathcal{D}^*,H_c,T_c\mathcal{D}^*)\end{aligned}\right\} \tag{34}$$

when using the dimensional constants

$$\left.\begin{aligned}&\Psi_c=\frac{M_c/(Da^3)}{2\Omega r_e},\ U_c=\frac{\lambda^2SM_c/(Da^3)}{(i\nu+\alpha)r_e}\\ &V_c=-\Psi_c,\ W_c=r_e^{-1}\Psi_c\\ &H_c=\frac{2\Omega r_eU_c}{\lambda_o^2},\ T_c=\frac{2\Omega r_eU_c}{\lambda^2R}\end{aligned}\right\}. \tag{35}$$

3. CALCULATED ZONAL CIRCULATION

STANDARD PARAMETERS USED

The latitudinal and vertical variation of the assumed sources has already been described [cf. eq (16) to (19), eq (24), and fig. 1]. The consequent two-dimensional structure of the heat source and momentum flux, respectively, is shown in figures 2 and 3. These sources are independent of time (i.e., of zero frequency), since we are considering the annual mean component of the circulation. The eddy momentum transport is maximum

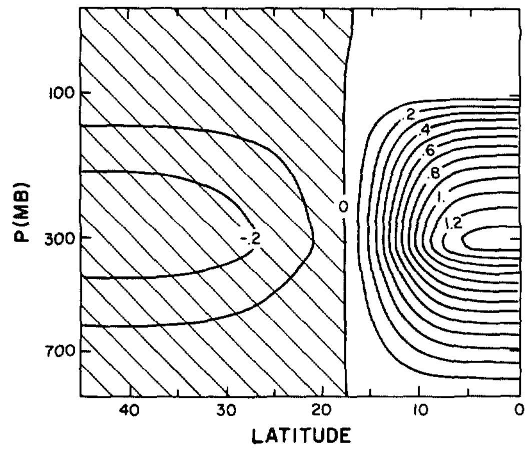

FIGURE 2.—Contours of the assumed zonal mean heating (in deg./day). The total heat input in a vertical column at the Equator is 50 kilolangleys/yr.

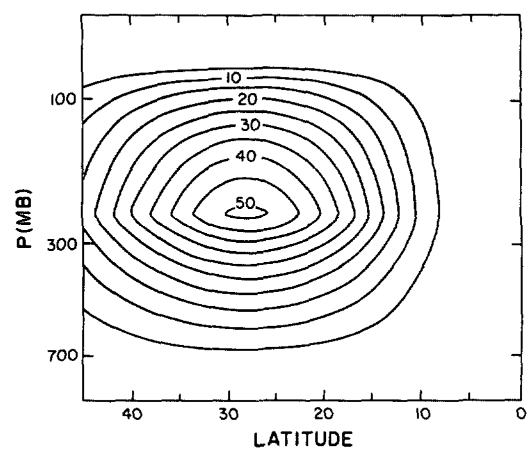

FIGURE 3.—Contours of the assumed zonal mean eddy momentum transport (in m^2/s^2).

in the upper troposphere with maximum amplitude corresponding roughly to that observed. The magnitude of the vertically integrated heat input was adjusted until the specified momentum fluxes largely extracted the westerly momentum provided by the Hadley circulation in the upper troposphere of the subtropics. The column heat input so obtained is comparable to that deduced from the part of the observed annual mean rainfall in the Tropics symmetric about the Equator. The maximum amplitude at the Equator is 50 kilolangleys/yr.

We took $(19.4\text{ day})^{-1}$ as the value for the Newtonian cooling and Rayleigh drag coefficients. A damping time of $\sim$20 days is chosen for the reasons discussed in the previous section. The value of S used is 0.005 corresponding to a lapse rate 0.76 of the dry adiabat at a temperature of 220°K. This S is appropriate to the tropical upper troposphere (cf. fig. 4 of Dickinson 1969).

RESULTS OF THE CALCULATIONS

First, we discuss the computed motions for thermal forcing alone and then the circulation obtained when Reynolds stresses are also present. Figure 4 shows the response to the given thermal source. That is, the air

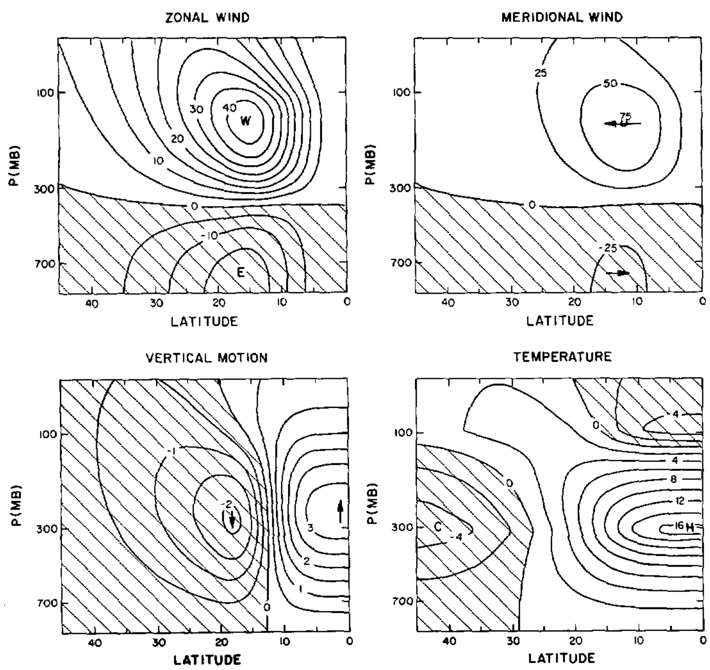

FIGURE 4.—Response to the assumed thermal forcing; (upper left) zonal wind (in m/s); (upper right) meridional wind (in cm/s); (lower left) vertical motion (in mm/s); (lower right) perturbation temperature (°C).

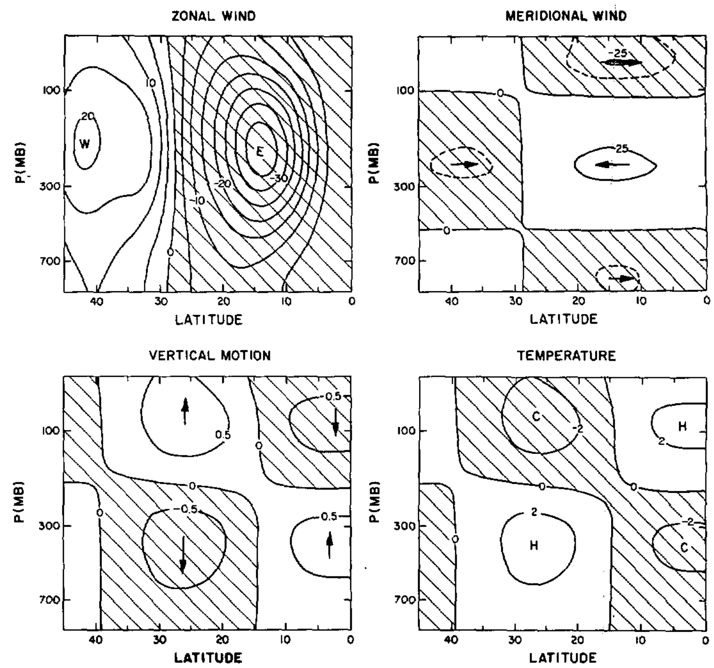

FIGURE 6.—Response to the assumed eddy momentum transport. The contours are the same as those for figure 4.

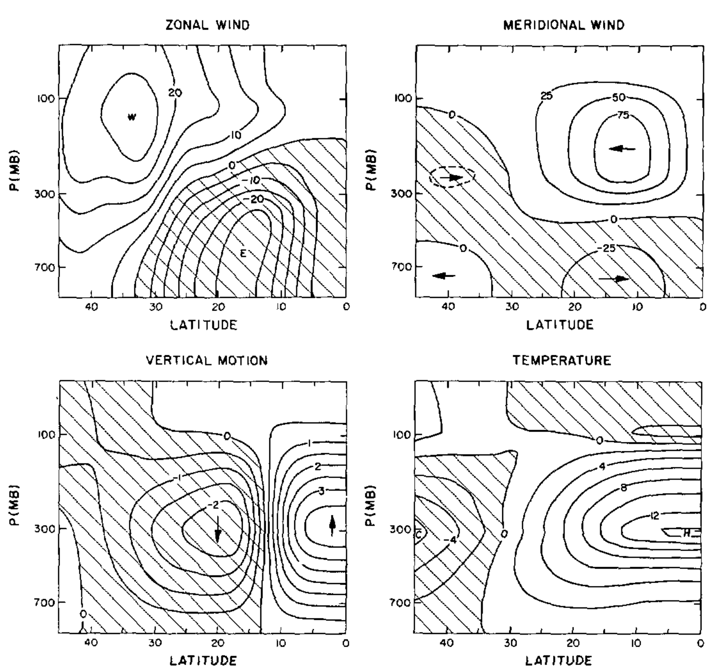

FIGURE 5.—Response to the sum of the assumed thermal forcing and eddy momentum transports. The contours are the same as those for figure 4.

rises at the Equator, flows poleward, and sinks in the subtropics. The temperatures are interpreted as departures from hemispheric mean temperature at a given pressure level. Notable features include the temperature minimum at the Equator directly above the level of maximum heating in the upper troposphere. This tropopause results from the adiabatic cooling in the upward branch of the thermally driven Hadley cell. The maximum net cooling occurs right above the source region. The maximum horizontal gradient of heating in the Tropics gives maximum thermal winds at about 15° of latitude. The resulting westerly jet at this latitude in the upper troposphere is maintained against frictional loss of momentum by the Coriolis torque, which is maximum at that latitude. Likewise, the easterly trades are maintained by the negative Coriolis torque concomitant with the return branch of the Hadley cell. The surface easterlies extend into the middle latitudes, consistent with the single-cell circulation and with the neglect of surface friction.

Figure 5 shows the joint response to the annual mean thermal and momentum sources. The momentum divergence in the Tropics increases the strength of the poleward Hadley-cell meridional circulation in the upper troposphere. In middle latitudes in the troposphere, the momentum convergence forces a Ferrel cell (equatorward flow). The most dramatic change from the circulation produced only by thermal sources is the poleward displacement of the westerly jets in the upper troposphere. This shift in the jet location by 20° of latitude is the result of the eddy transport to middle latitudes of momentum generated in the subtropics. The region of maximum poleward temperature gradient in the lower stratosphere has moved from the Tropics into middle latitudes, consistent with the thermal wind balance. The subtropical easterlies in the lower troposphere have been strengthened, and the westerlies in middle latitudes brought down to the surface.

That part of the circulation driven by eddy momentum transports is shown in figure 6. In the process of extracting momentum from the Tropics, the momentum divergence sets up a poleward meridional circulation in the upper troposphere which through Coriolis torques provides a significant fraction of the momentum lost to the eddies. Equatorward return flows above and below the source region extract an amount of momentum equal to that provided to the source region and thus drive easterly winds. In this fashion, momentum is transported vertically by the forced meridional circulation into the region of large eddy transport. The stratospheric return flow is probably overestimated by the model for reasons discussed in the next section.

4. DEPENDENCE ON THE DISSIPATION PARAMETERS

There is considerable uncertainty concerning the quantitative description of friction to be used for a zonal wind model. As stated by Lorenz (1967, p. 95), the zonal winds can be deduced from momentum balance only to the extent that they can be deduced from the field of friction. It is likewise clear that the temperature field can be deduced from the heat input only insofar as it can be deduced from the temperature-dependent cooling.

The most obvious defect in the calculated results is the excessive subtropical easterlies in the lower troposphere. Momentum is extracted from the zonal flow below the center of the Hadley cell by Coriolis torque and by eddy momentum divergence. The maximum velocity of the return flow of the Hadley cell is on the order of 40 cm/s, which is somewhat less than indicated by observations. The obvious conclusion is that, for a given wind speed, the rate of momentum exchange between the lower troposphere and the ground is much greater than that between the upper troposphere and the ground. The Coriolis torque adds no net momentum to a vertical column, and the momentum transports extract net momentum out of the latitudes considered. Hence, it is not possible to simulate the mid-latitude westerly jets in the upper troposphere with the assumed constant drag coefficient without the generation in the subtropical lower troposphere of winds of at least comparable amplitudes. We shall introduce in part II (Dickinson 1971) a variation of the drag coefficient with height to eliminate in part this difficulty.

The dependence on α and d for a time-independent source can be readily seen from eq (1) and (4) in the following limiting cases:

1. *Small d, thermal source.* As $d \rightarrow 0$, the meridional circulation vanishes since no Coriolis torque is required to maintain the mean wind. The temperature perturbation is then just that required to balance the heat input. The limiting zonal wind is determined from the temperature by the thermal wind relation.

2. *Small d, momentum source.* As $d \rightarrow 0$, the frictional loss of momentum vanishes, so momentum convergences are balanced entirely by Coriolis torque. The maximum possible meridional velocities occur in the source region. The meridional velocity is zero outside the source region. The temperatures are determined from the vertical motions accompanying this meridional circulation. The limiting strength of the zonal wind again follows from the temperature field by the thermal wind relation.

3. *Large d.* As $d \rightarrow \infty$, the wind and hence temperature perturbation vanish. For a momentum source, which is balanced by the drag, the meridional circulation also vanishes. For a given thermal source, the adiabatic cooling must increasingly balance the thermal source as d increases. Consequently, the thermally driven meridional circulation attains maximum strength as $d \rightarrow \infty$.

4. *Small α, thermal source.* As $\alpha \rightarrow 0$, the radiative damping vanishes. The thermal source is then balanced by adiabatic cooling, which determines the meridional circulation. The zonal winds follow from the Coriolis torques and the temperatures from the zonal winds by the thermal wind relation.

5. *Small α, momentum source.* Absence of radiative damping as $\alpha \rightarrow 0$ implies the vanishing of adiabatic warming, and hence there is no meridional circulation. The momentum source must be balanced entirely by drag. This balance determines the zonal winds and hence the temperature.

6. *Large α.* As $\alpha \rightarrow \infty$, the temperature perturbation and hence zonal wind vanish. A thermal source is balanced by radiative damping and hence does not produce a meridional circulation. Since in the absence of a zonal wind there is no drag, a momentum source is balanced entirely by Coriolis torques, thus determining a meridional circulation.

The above discussion is of general validity provided d and α are regarded as the inverse time scales associated with arbitrary flow-dependent momentum-loss processes and temperature-dependent radiative damping, respectively. A detailed scale analysis is necessary to determine which of the various limiting situations can be assumed for a first approximation. Periodic time-dependent flows are included if we add $i\nu$ to α and d. The various circulations then possible for $d=0$ were previously discussed (Dickinson 1968).

The details of meridional circulation above and below a region of momentum convergence are quite sensitive to the relative values of d and α. For a steady circulation with $d=0$, a vertical motion and temperature perturbation occur in the absence of a bottom boundary only below the source region (Dickinson 1968). It appears likely that the radiative damping in the lower stratosphere is faster than frictional damping, in which case the return flow would be expected to be largely below the source region. This would also be the case if d were much larger below than above the source region while α at different levels remained essentially constant. As an extreme case, all the return flow might occur in a frictional boundary layer. In view of these considerations, the return meridional cell indicated in figure 6 above the momentum source region is probably an overestimate of the cell actually forced by horizontal eddy momentum transports in the troposphere.

5. CONCLUDING REMARKS

The model we have described indicates how sources of heat and momentum jointly determine the zonal mean wind, temperature structure, and meridional circulation in the troposphere. These sources have been but crudely incorporated into our model. However, we anticipate a future need for similar but much more elaborate models integrated numerically, where every effort is made to include the proper distribution of sources and to relate these sources back to the zonal mean variables. Such models would be useful for studying certain aspects of climatic change. Before such an effort is made, however, it is well to understand where the greatest uncertainties in such an approach presently lie and where it might be possible to lessen these uncertainties. It is necessary to have a thorough knowledge of the different sources and sinks of heat, the redistribution of heat by the spectrum of eddy motions from small-scale turbulence to planetary waves, the distribution of radiative sources and sinks, and the distribution of latent heat by precipitation processes. It would be helpful to have an improved description of the zonal mean latent heat release as a function of time, height, and latitude. Even monthly averaged values for zonal mean rainfall in the Tropics are not available, and the parameterization of the vertical distribution of heating of the large-scale flow by tropical cumulus convection is still very crude. However, perhaps the greatest improvement could be introduced by replacing the Newtonian cooling approximation with a more realistic description of the heat losses which are dependent on the perturbation temperature. Likewise, the greatest improvement in the description of the momentum budget should result from a better description of the relationship to the mean flow not only of the drag by small-scale motions but also of the transport by large-scale eddies.

The analytic techniques of this paper can no longer be used for more realistic treatments. They are to be commended largely for the greater insight they offer into the dynamics of the idealized simple models considered here. For example, they help define the concept of the latitudinal scale of disturbances on the equatorial β-plane by associating this scale with the separation of variables parameter. The momentum divergences by large-scale eddies can be characterized by dominant latitudinal scales because of their broad oscillating distribution. On the other hand, the sharp equatorial maximum and broad minimum distributions of nonadiabatic heating require a very broad spectrum of individual normal modes for synthesis. It is thus erroneous to interpret the thickness of the tropical rainfall belt as an appropriate latitudinal scale for analysis of driven motions.

The conclusions derived from our model concerning the general circulation have all undoubtedly been stated previously, and some are quite familiar. The model serves to illustrate the interconnections among the various concepts such as the following: The difference between the atmospheric heating in equatorial latitudes by release of latent heat in the tropical rain belt and the temperature-dependent radiative cooling in the higher latitudes drives a Hadley circulation. The poleward branch of the Hadley circulation, in turn, drives a westerly jet in the upper troposphere in tropical latitudes; the adiabatic warming by the subsiding branch creates temperatures in excess of those required for radiative balance at latitudes well poleward of the region of large nonadiabatic heating. Consistent with the thermal wind balance, the maximum poleward temperature decrease maintained by the latent heat release is in tropical latitudes. The adiabatic cooling in the upward branch of the Hadley circulation at levels above the region of latent heat release forces the temperature to be less than the radiative equilibrium temperatures. The synoptic and planetary scale eddies transport westerly momentum from the tropical upper troposphere, where it is brought by the Hadley cell, into middle latitudes and, consequently, force an extensive poleward shift of the maximum westerlies. The divergence and convergence of momentum by the eddies in the upper troposphere force poleward and equatorward flows, respectively. These meridional flows, together with their return branches below, act to strengthen the Hadley circulation in the Tropics, produce a Ferrel circulation in middle latitudes, and in general redistribute momentum vertically from source to sink regions. The vertical branches of the component of the meridional circulation forced by the eddies must by adiabatic warming weaken the equatorward temperature gradient in the Tropics and strengthen the gradient in middle latitudes, consistent with the poleward displacement of the maximum westerlies. The relative contribution of thermal or momentum sources to a steady Hadley circulation depends on the details of the drag and the radiative damping. If momentum is damped at a much greater rate than is a temperature perturbation, the meridional circulation will be controlled largely by the distribution of heat sources. Conversely, if the momentum damping rate is small compared to that of temperature, the meridional circulation will be largely controlled by the distribution of momentum sources.

Our conclusion that the tropical rain belt by itself drives a Hadley cell of magnitude comparable to that observed if rates of momentum and radiative damping are of comparable amplitude may be contrasted with the results of Kuo (1956). He concludes that a latitudinal distribution of heating modeling the effect of solar radiation would produce a Hadley circulation of only a few centimeters per second and, therefore, that the observed tropical meridional cell must necessarily be driven primarily by the divergence of eddy momentum fluxes from the Tropics. A result similar to Kuo's would follow from our model if we considered only the response to a global scale distribution of heating. Heating put in "near" the lower boundary is ineffective in our model in driving meridional circulations because of the constraint of vanishing vertical velocity at the lower boundary. Global scale components of forcing, as assumed by Kuo, have small separation parameter λ and, consequently, a large vertical scale. Thus, even upper tropospheric sources are effectively close enough to the bottom boundary that a relatively small response is produced.

REFERENCES

Dickinson, Robert E., "On the Excitation and Propagation of Zonal Winds in an Atmosphere With Newtonian Cooling," *Journal of the Atmospheric Sciences*, Vol. 25, No. 2, Mar. **1968**, pp. 269–279.

Dickinson, Robert E., "Propagators of Atmospheric Motions: Part 1. Excitation by Point Impulses," *Reviews of Geophysics*, Vol. 7, No. 3, Aug. **1969**, pp. 483–514.

Dickinson, Robert E., "Analytic Model for Zonal Winds in the Tropics: II. Variation of the Tropospheric Mean Structure With Season and Differences Between Hemispheres," *Monthly Weather Review*, Vol. 99, No. 6, June **1971**, pp. 511–523.

Dickinson, Robert E., and Geller, Marvin A., "A Generalization of 'Tidal Theory With Newtonian Cooling,' " *Journal of the Atmospheric Sciences*, Vol. 25, No. 5, Sept. **1968**, pp. 932–933.

Eliassen, Arnt, "Slow Thermally or Frictionally Controlled Meridional Circulations in a Circular Vortex," *Astrophysica Norvegica*, Vol. 5, No. 2, Oslo, Norway, May **1952**, pp. 19–60.

Gilman, Peter A., "On the Mean Meridional Circulation in the Presence of a Steady State, Symmetric, Circumpolar Vortex," *Tellus*, Vol. 16, No. 2, Stockholm, Sweden, May **1964**, pp. 160–167.

Holton, James R., "A Note on the Propagation of the Biennial Oscillation," *Journal of the Atmospheric Sciences*, Vol. 25, No. 3, May **1968**, pp. 519–521.

Kung, Ernest C., "Diurnal and Long-Term Variations of the Kinetic Energy Generation and Dissipation for a Five-Year Period," *Monthly Weather Review*, Vol. 95, No. 9, Sept. **1967**, pp. 593–606.

Kuo, H.-L., "Forced and Free Meridional Circulations in the Atmosphere," *Journal of Meteorology*, Vol. 13, No. 6, Dec. **1956**, pp. 561–568.

Kuo, H.-L., "On Formation and Intensification of Tropical Cyclones Through Latent Heat Release by Cumulus Convection," *Journal of the Atmospheric Sciences*, Vol. 22, No. 1, Jan. **1965**, pp. 40–63.

Leovy, Conway, "Simple Models of Thermally Driven Mesospheric Circulation," *Journal of the Atmospheric Sciences*, Vol. 21, No. 4, July **1964**, pp. 327–341.

Lorenz, Edward N., "The Nature and Theory of the General Circulation of the Atmosphere," *World Meteorological Organization Technical Note* No. 218, TP 115, Geneva, Switzerland, **1967**, 161 pp.

Manabe, Syukuro, and Strickler, Robert F., "Thermal Equilibrium of the Atmosphere With a Convective Adjustment," *Journal of the Atmospheric Sciences*, Vol. 21, No. 4, July **1964**, pp. 361–385.

Oort, Abraham H., "On Estimates of the Atmospheric Energy Cycle," *Monthly Weather Review*, Vol. 92, No. 11, Nov. **1964**, pp. 483–493.

Riehl, Herbert, and Malkus, Joanne S., "On the Heat Balance in the Equatorial Trough Zone," *Geophysica*, Vol. 6, No. 3/4, Helsinki, Finland, **1958**, pp. 503–538.

Wiin-Nielsen, A., Vernekar, A., and Yang, C. H., "On the Development of Baroclinic Waves Influenced by Friction and Heating," *Pure and Applied Geophysics*, Vol. 68, No. 3, Basel, Switzerland, **1967**, pp. 131–161.

[Received June 8, 1970; revised August 3, 1970]